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Bistatic Radar Cross Sections of
Horizontally Oriented Chaff

Peyton Z. Peebles, Jr.

March 1984

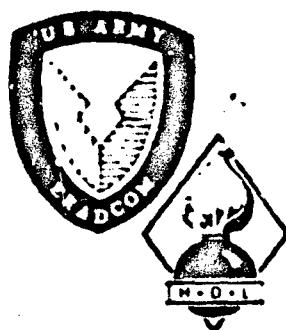
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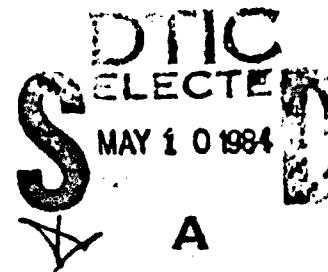


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Radar chaff	Dipoles											
20. ABSTRACT (Continue on reverse side if necessary and identify by block number) A Bistatic Radar Cross sections are determined for scattering from a cloud of randomly positioned resonant dipoles (chaff). Dipoles are assumed to be horizontally oriented with axes randomly oriented in the horizontal plane. The cloud is arbitrarily located relative to an illuminating source having an arbitrary (elliptical) polarization. Cloud cross section is found for an arbitrarily located receiver that views the cloud with an antenna of arbitrary polarization. A cross section applicable to the receiver's orthogonal polarization is also found.												

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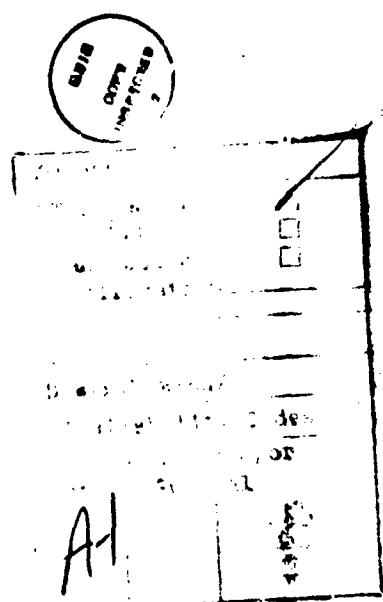
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1. INTRODUCTION

In earlier work^{1,2} bistatic radar cross sections were determined for scattering from a cloud of randomly positioned resonant dipoles (chaff) having axes randomly and uniformly distributed in direction over a sphere. In this paper we again consider scattering from a cloud of randomly positioned dipoles, but extend the earlier work to the case where dipole axes all lie in a horizontal plane with random and uniform distribution of directions within the plane. For some practical chaff the horizontal distribution is more realistic than the spherical distribution.

The geometry applicable to a typical dipole is shown in figure 1. The axis of the dipole has its midpoint at D, the origin of coordinate system x' , y' , z' . Point D is in the coordinate frame x , y , z with spherical coordinates (r_1, θ_1, ϕ_1) . A transmitter at point T radiates an arbitrarily polarized wave toward D. A receiver at point R, located at (r_2, θ_2, ϕ_2) in spherical coordinates within the x' , y' , z' frame, receives bistatic scattering from the dipole. The receiver is presumed to have a preferred, but arbitrary, polarization that can be different than that of the transmitter. Axes of the x' , y' , z' frame are parallel, respectively, to those of the x , y , z frame. The axis of the dipole is assumed to lie in the x' , y' plane and form an angle ϕ_d from the x' axis. The angle ϕ_d is assumed random with uniform distribution on $(0, 2\pi)$.

More generally, for fixed points T and R, scattering at R is due to many dipoles in a cloud. These dipoles are assumed to be distributed randomly and uniformly in position so that the cross sections seen at point R become the cross sections of a single dipole multiplied by N, the number of common dipoles illuminated by T and viewed by the receiver at R.^{1,2} Other assumptions leading to this result are given in earlier work.^{1,2,*} Thus, only a single dipole requires analysis.

¹ Peebles, Peyton Z., Jr., Bistatic Radar Cross Sections of Chaff, IEEE Trans. Aerosp. Electron. Syst., AES-20, No. 2 (March 1984).

² Peebles, Peyton Z., Jr., Bistatic Radar Cross Sections of Chaff, University of Florida, Electronic Communications Laboratory, prepared the report for Harry Diamond Laboratories, HDL-CR-83-107-6 (June 1983).

* Mainly, r_1 and r_2 are large enough that they have approximately the same values for all dipoles of interest (those in the common volume of intersection of transmit and receive antenna patterns).

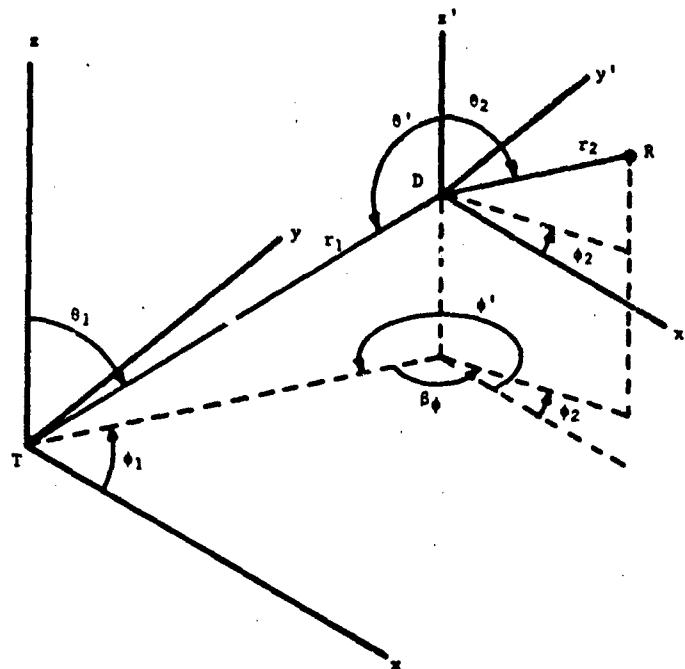


Figure 1. Geometry of bistatic scattering. A transmitter is located at point T, a receiver is at R and the scattering dipole is at point D.

2. ANALYSIS

Unfortunately, for the planar dipole distribution there seems to be no simple way to separate dipole scattering from transmit-receive station geometry, as was done in earlier work¹ using the scattering plane approach for a spherical dipole distribution. Because of this fact a direct approach to analysis is indicated. As a consequence, the resulting cross-section formulas are somewhat more cumbersome than for the spherical distribution, but can still be obtained.

2.1 General Equations

Consider first a very thin highly conducting dipole of total physical length L, having its wire axis pointing in the direction (θ_d, ϕ_d) in spherical coordinates. It is helpful to think of the dipole as located at point D in figure 1. Let P be an arbitrary point at (r, θ, ϕ) from the dipole in spherical coordinates. If the dipole radiates due to excitation by a terminal current

¹

Peebles, Peyton Z., Jr., Bistatic Radar Cross Sections of Chaff, IEFE Trans. Aerosp. Electron. Syst., AES-20, No. 2 (March 1984).

I_T , the electric field components E_θ and E_ϕ in directions θ and ϕ at P are known.³ In matrix notation they may be written as

$$\begin{bmatrix} E_\theta \\ E_\phi \end{bmatrix} = \frac{-jn e^{j[\omega t - (2\pi r/\lambda)]}}{2\lambda r} \begin{bmatrix} h_\theta \\ h_\phi \end{bmatrix} I_T , \quad (1)$$

where n is the intrinsic impedance of the medium ($n = 120\pi$ for free space), $j = \sqrt{-1}$, ω is angular frequency, λ is wavelength, t is time, and

$$\begin{bmatrix} h_\theta \\ h_\phi \end{bmatrix} = A \begin{bmatrix} \cos \theta \sin \theta_d \cos(\phi - \phi_d) - \sin \theta \cos \theta_d \\ \sin \theta_d \sin(\phi_d - \phi) \end{bmatrix} , \quad (2)$$

$$A = \frac{(\lambda/\pi)}{\sin(\pi L/\lambda)} \cdot \frac{\cos\left[\frac{\pi L}{\lambda} \cos \psi\right] - \cos(\pi L/\lambda)}{\sin^2 \psi} , \quad (3)$$

$$\cos \psi = \cos \theta \cos \theta_d + \sin \theta \sin \theta_d \cos(\phi - \phi_d) . \quad (4)$$

Here h_θ and h_ϕ are the effective lengths of the dipole evaluated in the direction (θ, ϕ) .

The current I_T that excites the fields of equation (1) is induced by fields at D that are presumed to be due to a source in a direction possibly different from that of point P (actually due to the transmitter at T in figure 1). If (θ', ϕ') is the direction of the source, the terminal current becomes

$$I_T = \frac{1}{Z_{rad}} \begin{bmatrix} h_\theta' & h_\phi' \end{bmatrix} \begin{bmatrix} E_\theta' \\ E_\phi' \end{bmatrix} , \quad (5)$$

where Z_{rad} is the dipole's radiation impedance, E_θ' and E_ϕ' are field components at the dipole in directions θ' and ϕ' , respectively, and h_θ' and h_ϕ' are equal to h_θ and h_ϕ of equation (2) evaluated for $\theta = \theta'$, $\phi = \phi'$.

³

Cross, J. L., Response of Arrays to Stochastic Fields, Ph.D. dissertation, University of Florida (1969).

2.2 Special Equations

For the problem at hand, θ' and ϕ' define the location of the transmitter relative to the dipole in figure 1, while θ and ϕ define the receiver's location. Thus, we set $\theta = \theta_2$, $\phi = \phi_2$, $E_\theta = E_{\theta_2}$, $E_\phi = E_{\phi_2}$, $\theta' = \pi - \theta_1$, $\phi' = \pi + \phi_1$, $E_{\theta'} = E_{\theta_1}$, and $E_{\phi'} = -E_{\phi_1}$ in the general equations. Here we define E_{θ_1} and E_{ϕ_1} as electric field components at D in directions θ_1 and ϕ_1 , respectively, due to the transmitter at T. By using the additional fact that $\theta_d = \pi/2$ for the dipoles of interest here, we substitute equation (5) into (1) to obtain

$$\begin{bmatrix} E_{\theta_2} \\ E_{\phi_2} \end{bmatrix} = \begin{bmatrix} d_{11} & d_{12} \\ d_{21} & d_{22} \end{bmatrix} \begin{bmatrix} E_{\theta_1} \\ E_{\phi_1} \end{bmatrix} B e^{j\omega t}, \quad (6)$$

where

$$B \triangleq \frac{-j2\pi r_2/\lambda}{2\lambda Z_{rad} r_2}, \quad (7)$$

$$d_{11} = A_1 A_0 \cos \theta_1 \cos \theta_2 \cos(\phi_1 - \phi_d) \cos(\phi_2 - \phi_d), \quad (8a)$$

$$d_{12} = -A_1 A_0 \cos \theta_2 \sin(\phi_1 - \phi_d) \cos(\phi_2 - \phi_d), \quad (8b)$$

$$d_{21} = -A_1 A_0 \cos \theta_1 \cos(\phi_1 - \phi_d) \sin(\phi_2 - \phi_d), \quad (8c)$$

$$d_{22} = A_1 A_0 \sin(\phi_1 - \phi_d) \sin(\phi_2 - \phi_d), \quad (8d)$$

and

$$A_0 = \frac{\cos\left[\frac{\pi L}{\lambda} \cos \psi_1\right] - \cos\left(\frac{\pi L}{\lambda}\right)}{\sin^2 \psi_1} \cdot \frac{\cos\left[\frac{\pi L}{\lambda} \cos \psi_2\right] - \cos\left(\frac{\pi L}{\lambda}\right)}{\sin^2 \psi_2}, \quad (9)$$

$$A_1 = (\lambda/\pi)^2 / \sin^2(\pi L/\lambda), \quad (10)$$

with

$$\cos \psi_1 = -\sin \theta_1 \cos(\phi_1 - \phi_d), \quad (11)$$

$$\cos \psi_2 = \sin \theta_2 \cos(\phi_2 - \phi_d). \quad (12)$$

2.3 Cross Sections

We only briefly outline the development of cross section formulas because the procedures follow those in the earlier work.¹ The total received electric-field vector, denoted by \vec{E}_R , can be decomposed into two orthogonally polarized components \vec{E}_{R_1} and \vec{E}_{R_2} , that have "amplitudes" E_{R_1} and E_{R_2} , respectively. \vec{E}_R , has the arbitrary polarization of the receiver that is determined by the receiver's field component ratio, Q_R .¹ The power in \vec{E}_{R_1} is proportional to

$$|\vec{E}_{R_1}|^2 = (1 + |Q_R|^2)|E_{R_1}|^2 . \quad (13)$$

Similarly, the power in the orthogonally polarized field component is proportional to

$$|\vec{E}_{R_2}|^2 = (1 + |Q_R|^2)|E_{R_2}|^2 . \quad (14)$$

Furthermore,¹

$$\begin{bmatrix} E_{R_1} \\ E_{R_2} \end{bmatrix} = \frac{1}{1 + |Q_R|^2} \begin{bmatrix} 1 & Q_R^* \\ -Q_R & 1 \end{bmatrix} \begin{bmatrix} E_{\theta_2} \\ E_{\phi_2} \end{bmatrix} , \quad (15)$$

where * represents complex conjugation.

In an analogous manner, the field components E_{θ_1} and E_{ϕ_1} are related to the transmitter's field component ratio, denoted as Q_T , by¹

$$\begin{bmatrix} E_{\theta_1} \\ E_{\phi_1} \end{bmatrix} = \begin{bmatrix} 1 \\ Q_T \end{bmatrix} E_T , \quad (16)$$

where E_T is the complex "amplitude" of the electric field vector, denoted by \vec{E}_I , incident on the dipole. By substituting equations (6) and (16) into (15) we have

¹ Peebles, Peyton Z., Jr., Bistatic Radar Cross Sections of Chaff, IEEE Trans. Aerosp. Electron. Syst., AES-20, No. 2 (March 1984).

$$\begin{bmatrix} E_{R_1} \\ E_{R_2} \end{bmatrix} = \frac{B e^{j\omega t}}{1 + |Q_R|^2} \begin{bmatrix} 1 & Q_R^* \\ -Q_R & 1 \end{bmatrix} \begin{bmatrix} d_{11} & d_{12} \\ d_{21} & d_{22} \end{bmatrix} \begin{bmatrix} 1 \\ Q_T \end{bmatrix} E_T . \quad (17)$$

Next, as in the earlier work,¹ we define average cross sections by

$$\bar{\sigma} = 4\pi r_2^2 E[|\vec{E}_{R_1}|^2]/|\vec{E}_1|^2 = \frac{4\pi r_2^2 (1 + |Q_R|^2) E[|E_{R_1}|^2]}{(1 + |Q_T|^2) |E_T|^2} , \quad (18)$$

$$\bar{\sigma}_x = 4\pi r_2^2 E[|\vec{E}_{R_2}|^2]/|\vec{E}_1|^2 = \frac{4\pi r_2^2 (1 + |Q_R|^2) E[|E_{R_2}|^2]}{(1 + |Q_T|^2) |E_T|^2} , \quad (19)$$

where $E[\cdot]$ represents the statistical expectation operation, and r_2 is assumed large. The second forms of equations (15) and (16) derive from the use of equations (13) and (14), and the fact that

$$|\vec{E}_1|^2 = (1 + |Q_T|^2) |E_T|^2 . \quad (20)$$

Solutions for $\bar{\sigma}$ and $\bar{\sigma}_x$ follow from solving equation (17) for E_{R_1} and E_{R_2} and substituting these quantities into equations (18) and (19). The expectations involved are each found to contain 16 terms of the form $d_{ij}d_{mn}^*$. Examination of these terms, using equation (8), shows that some terms are equal according to the following definitions:

¹ Peebles, Peyton Z., Jr., Bistatic Radar Cross Sections of Chaff, IEEE Trans. Aerosp. Electron. Syst., AES-20, No. 2 (March 1984).

$$\begin{aligned}
z_1 &\triangleq d_{11}d_{11}^*, \\
z_2 &\triangleq d_{11}d_{12}^* = d_{12}d_{11}^*, \\
z_3 &\triangleq d_{12}d_{12}^*, \\
z_4 &\triangleq d_{21}d_{21}^*, \\
z_5 &\triangleq d_{21}d_{22}^* = d_{22}d_{21}^*, \\
z_6 &\triangleq d_{22}d_{22}^*, \\
z_7 &\triangleq d_{11}d_{21}^* = d_{21}d_{11}^*, \\
z_8 &\triangleq d_{11}d_{22}^* = d_{12}d_{21}^* = d_{21}d_{12}^* + d_{22}d_{11}^*, \\
z_9 &\triangleq d_{12}d_{22}^* = d_{22}d_{12}^*.
\end{aligned} \tag{21}$$

The number of distinct parameters is therefore reduced from 16 to 9 which makes the solutions of equations (18) and (19) somewhat simpler. Note, however, that nine parameters are now required to define cross sections, whereas only four were necessary when dipoles are spherically distributed as in the earlier work.¹

If parameters σ_i are defined according to

$$\sigma_i \triangleq 4\pi r_2^2 E[|B|^2 z_i], \quad i = 1, 2, \dots, 9, \tag{22}$$

the solutions for the cross sections can be written as

$$\begin{aligned}
\delta = \frac{1}{(1 + |Q_T|^2)(1 + |Q_R|^2)} & \left\{ [\sigma_1 + 2\sigma_2 \operatorname{Re}(Q_T) + \sigma_3 |Q_T|^2] \right. \\
& + 2 \operatorname{Re}(Q_R) [\sigma_7 + 2\sigma_8 \operatorname{Re}(Q_T) + \sigma_9 |Q_T|^2] \\
& \left. + |Q_R|^2 [\sigma_4 + 2\sigma_5 \operatorname{Re}(Q_T) + \sigma_6 |Q_T|^2] \right\}, \tag{23}
\end{aligned}$$

¹ Peebles, Peyton Z., Jr., Bistatic Radar Cross Sections of Chaff, IEEE Trans. Aerosp. Electron. Syst., AES-20, No. 2 (March 1984).

$$\delta_x = \frac{1}{(1 + |Q_T|^2)(1 + |Q_R|^2)} \left\{ \begin{aligned} & [\sigma_4 + 2\sigma_5 \operatorname{Re}(Q_T) + \sigma_6 |Q_T|^2] \\ & - 2 \operatorname{Re}(Q_R) [\sigma_7 + 2\sigma_8 \operatorname{Re}(Q_T) + \sigma_9 |Q_T|^2] \\ & + |Q_R|^2 [\sigma_1 + 2\sigma_2 \operatorname{Re}(Q_T) + \sigma_3 |Q_T|^2] \end{aligned} \right\} . \quad (24)$$

Here $\operatorname{Re}[\cdot]$ represents the real part of the bracketed quantity. Specific expressions for the parameters σ_i derive from equation (22) using equations (21) and (7). They are

$$\sigma_1/\lambda^2 = A_a \int_0^{2\pi} A_0^2 \cos^2 \phi_d \cos^2(\phi_d - \beta_\phi) d\phi_d \cos^2 \theta_1 \cos^2 \theta_2 , \quad (25a)$$

$$\sigma_2/\lambda^2 = A_a \int_0^{2\pi} A_0^2 \cos \phi_d \sin \phi_d \cos^2(\phi_d - \beta_\phi) d\phi_d \cos \theta_1 \cos^2 \theta_2 , \quad (25b)$$

$$\sigma_3/\lambda^2 = A_a \int_0^{2\pi} A_0^2 \sin^2 \phi_d \cos^2(\phi_d - \beta_\phi) d\phi_d \cos^2 \theta_2 , \quad (25c)$$

$$\sigma_4/\lambda^2 = A_a \int_0^{2\pi} A_0^2 \cos^2 \phi_d \sin^2(\phi_d - \beta_\phi) d\phi_d \cos^2 \theta_1 , \quad (25d)$$

$$\sigma_5/\lambda^2 = A_a \int_0^{2\pi} A_0^2 \sin \phi_d \cos \phi_d \sin^2(\phi_d - \beta_\phi) d\phi_d \cos \theta_1 , \quad (25e)$$

$$\sigma_6/\lambda^2 = A_a \int_0^{2\pi} A_0^2 \sin^2 \phi_d \sin^2(\phi_d - \beta_\phi) d\phi_d , \quad (25f)$$

$$\sigma_7/\lambda^2 = -A_a \int_0^{2\pi} A_0^2 \cos^2 \phi_d \cos(\phi_d - \beta_\phi) \sin(\phi_d - \beta_\phi) d\phi_d \cos^2 \theta_1 \cos \theta_2 , \quad (25g)$$

$$\sigma_8/\lambda^2 = -A_a \int_0^{2\pi} A_0^2 \cos \phi_d \sin \phi_d \cos(\phi_d - \beta_\phi) \sin(\phi_d - \beta_\phi) d\phi_d \cos \theta_1 \cos \theta_2 , \quad (25h)$$

$$\sigma_9/\lambda^2 = -A_a \int_0^{2\pi} A_0^2 \sin^2 \phi_d \cos(\phi_d - \beta_\phi) \sin(\phi_d - \beta_\phi) d\phi_d \cos \theta_2 , \quad (25i)$$

where we define

$$A_a \triangleq [n/\sqrt{2} Z_{rad} \pi^2 \sin^2(\pi L/\lambda)]^2 , \quad (26)$$

$$\beta_\phi \triangleq \pi + \phi_2 - \phi_1 . \quad (27)$$

2.4 Coefficient Evaluation

To use equations (23) or (24) the coefficients of (25) must be evaluated. Because of the complexity of the integrands, mainly due to A_0 of (9), closed solutions for the integrals were not obtained. Solutions were obtained, however, using a digital computer. Coefficients depend on the three variables θ_1 , θ_2 , and β_ϕ , once a particular relative dipole length is chosen (L/λ sets Z_{rad} , see the earlier paper¹). The symmetry of (25) as a function of θ_1 or θ_2 can be analytically determined. Symmetry of (25) with β_ϕ was determined by computer. Table I gives a summary of results.

TABLE I. SYMMETRY OF COEFFICIENTS WITH VARIATIONS IN θ_1 , θ_2 , and β_ϕ

i in σ_i	Symmetry [†] of σ_i about			
	$\theta_1 = \pi/2$	$\theta_2 = \pi/2$	$\beta_\phi = 0$	$\beta_\phi = \pi/2$
1	E	E	E	E
2	O	E	O	O
3	E	E	E	E
4	E	E	E	E
5	O	E	O	O
6	E	E	E	E
7	E	O	O	O
8	O	O	E	E
9	E	O	O	O

[†] E = even, O = odd

A consequence of the results shown in Table I is that parameters σ_i need only be evaluated for each of the three variables θ_1 , θ_2 , and β_ϕ over a 90-degree range; we choose the ranges $0 \leq \theta_1 \leq \pi/2$, $0 \leq \theta_2 \leq \pi/2$, and $0 \leq \beta_\phi \leq \pi/2$. Tables 2, 3, and 4 give the computed results for half-wave ($L = \lambda/2$), full-wave ($L = \lambda$), and three halves-wave ($L = 3\lambda/2$) dipoles, respectively.

Several checks were made to verify the correctness of the computed data. For example, the backscatter point for horizontal transmit and receive polar-

¹ Peebles, Peyton Z., Jr., Bistatic Radar Cross Sections of Chaff, IEEE Trans. Aerosp. Electron. Syst., AES-20, No. 2 (March 1984).

izations with $\theta_1 = \pi/2$ and $\theta_2 = \pi/2$ can be solved analytically with $L = \lambda/2$. In this case only σ_6 is nonzero. The average backscatter cross section was found by Bloch, Hammermesh, and Phillips⁴ to be $\bar{\sigma} = 0.289\lambda^2$. Solution of $\bar{\sigma}$ as given in equation (23) gives $\bar{\sigma} = \sigma_6 = 0.2797\lambda^2$ where an integral given by Bloch et al.⁴ was used. The computed data give $\bar{\sigma} = \sigma_6 = 0.2795\lambda^2$ for an error of about 0.072 percent.

3. CROSS SECTIONS FOR SPECIAL CASES

Cross sections, as obtained from equation (23), can be obtained for a number of special cases of transmit and receive polarizations. We shall use subscripts on $\bar{\sigma}$ to indicate polarizations involved. The first subscript indicates the transmitter's polarization while the second applies to the wave's polarization at the receiver. We use V and H to represent linear polarizations where electric-field components are only vertical (θ_1 or θ_2 directions) and horizontal (ϕ_1 or ϕ_2 directions). Thus $\bar{\sigma}_{VH}$ corresponds to cross section as seen by the receiver when the transmitter transmits linear polarization only in the θ_1 (or V) direction and the receiver responds only to the linear field component in the ϕ_2 (or H) direction.

In an analogous manner 0 is used to represent circular polarization (sense, left and right, will be seen to be irrelevant). Linear polarizations tilted 45 degrees from the ϕ direction (ϕ_1 or ϕ_2) are denoted by /, while the opposite tilt for -45 degrees is denoted by subscript \.

3.1 Vertical and Horizontal Polarizations

To evaluate $\bar{\sigma}_{VV}$ the proper values of Q_T and Q_R are both zero from the earlier work.¹ By use of equation (23) we have

$$\bar{\sigma}_{VV} = \sigma_1 . \quad (28)$$

For $\bar{\sigma}_{HH}$ we use $Q_T = \infty$ and $Q_R = \infty$ to obtain

$$\bar{\sigma}_{HH} = \sigma_6 . \quad (29)$$

¹ Peebles, Peyton Z., Jr., Bistatic Radar Cross Sections of Chaff, IEEE Trans. Aerosp. Electron. Syst., AES-20, No. 2 (March 1984).

⁴ Bloch, F., M. Hammermesh, and M. Phillips, Return Cross Sections from Random Oriented Resonant Half-Wave Length Chaff, Harvard University, Radio Research Laboratory, Technical Memorandum 411-TM-127 (June 19, 1944).

For $\bar{\sigma}_{HV}$, $Q_T = \infty$, $Q_R = 0$, and

$$\bar{\sigma}_{HV} = \sigma_3 . \quad (30)$$

Similarly,

$$\bar{\sigma}_{VH} = \sigma_4 . \quad (31)$$

In these four simple cases only one of the parameters σ_i is needed to determine cross section. We note that $\bar{\sigma}_{VH} \neq \bar{\sigma}_{HV}$ in general.

3.2 Circular Polarizations

For circular polarizations we have $Q_T = \pm j$ and $Q_R = \pm j$ from the earlier paper.¹ Choice of sign determines rotation sense. Since $\text{Re}(Q_T) = 0$ and $\text{Re}(Q_R) = 0$ while $|Q_T|^2 = 1$ and $|Q_R|^2 = 1$, regardless of sense, equation (23) readily gives

$$\bar{\sigma}_{00} = (\sigma_1 + \sigma_3 + \sigma_4 + \sigma_6)/4. \quad (32)$$

Thus, the cross section seen by the receiver does not depend on the senses of either receiver or transmitter circular polarizations. This fact was also found earlier to be true for spherically distributed dipoles.¹

3.3 Tilted Linear Polarizations

When the linear polarization is tilted 45 degrees ($/$ notation) or -45 degrees (\backslash notation) from the appropriate ϕ direction (ϕ_1 or ϕ_2) the proper values of Q (Q_T or Q_R) are 1 or -1 , respectively. On substituting these values into (23) we obtain

$$\sigma_{//} = [(\sigma_1 + 2\sigma_2 + \sigma_3) + 2(\sigma_7 + 2\sigma_8 + \sigma_9) + (\sigma_4 + 2\sigma_5 + \sigma_6)]/4 , \quad (33)$$

$$\sigma_{/\backslash} = [(\sigma_1 + 2\sigma_2 + \sigma_3) - 2(\sigma_7 + 2\sigma_8 + \sigma_9) + (\sigma_4 + 2\sigma_5 + \sigma_6)]/4 , \quad (34)$$

$$\sigma_{\backslash/} = [(\sigma_1 - 2\sigma_2 + \sigma_3) - 2(\sigma_7 - 2\sigma_8 + \sigma_9) + (\sigma_4 - 2\sigma_5 + \sigma_6)]/4 , \quad (35)$$

$$\sigma_{\backslash\backslash} = [(\sigma_1 - 2\sigma_2 + \sigma_3) + 2(\sigma_7 - 2\sigma_8 + \sigma_9) + (\sigma_4 - 2\sigma_5 + \sigma_6)]/4 . \quad (36)$$

We see that these linear polarization combinations require all nine parameters σ_i .

¹

Peebles, Peyton Z., Jr., Bistatic Radar Cross Sections of Chaff, IEEE Trans. Aerosp. Electron. Syst., AES-20, No. 2 (March 1984).

By solving equations (33) through (36) we may solve for σ_8 :

$$\sigma_8 = [(\sigma_{//} - \sigma_{\backslash\backslash})/4] + [(\sigma_{\backslash\backslash} - \sigma_{//})/4] . \quad (37)$$

Here σ_8 is somewhat analogous to σ_A in the earlier paper.¹

3.4 Other Combinations of Linear Polarizations

By proceeding in the same manner for various combinations of vertical, horizontal, and 45-degree tilted linear polarizations we have

$$\sigma_{VV} = (\sigma_1 + 2\sigma_7 + \sigma_4)/2 , \quad (38)$$

$$\sigma_{V\backslash} = (\sigma_1 - 2\sigma_7 + \sigma_4)/2 , \quad (39)$$

$$\sigma_{\backslash V} = (\sigma_1 + 2\sigma_2 + \sigma_3)/2 , \quad (40)$$

$$\sigma_{\backslash\backslash V} = (\sigma_1 - 2\sigma_2 + \sigma_3)/2 , \quad (41)$$

$$\sigma_{H\backslash} = (\sigma_3 + 2\sigma_9 + \sigma_6)/2 , \quad (42)$$

$$\sigma_{H\backslash\backslash} = (\sigma_3 - 2\sigma_9 + \sigma_6)/2 , \quad (43)$$

$$\sigma_{\backslash H} = (\sigma_4 + 2\sigma_5 + \sigma_6)/2 , \quad (44)$$

$$\sigma_{\backslash\backslash H} = (\sigma_4 - 2\sigma_5 + \sigma_6)/2 . \quad (45)$$

One of the advantages of developing equations (38) through (45) is that all remaining parameters σ_i can be derived as follows:

$$\sigma_2 = (\sigma_{VV} - \sigma_{\backslash V})/2 , \quad (46)$$

$$\sigma_5 = (\sigma_{\backslash H} - \sigma_{\backslash\backslash H})/2 , \quad (47)$$

$$\sigma_7 = (\sigma_{VV} - \sigma_{V\backslash})/2 , \quad (48)$$

$$\sigma_9 = (\sigma_{H\backslash} - \sigma_{H\backslash\backslash})/2 . \quad (49)$$

4. SUMMARY

An analysis has been given for bistatic radar scattering from a chaff cloud consisting of resonant dipoles randomly positioned in space in a uniform manner. Dipole axes were assumed to lie in a horizontal plane with random,

¹

Peebles, Peyton Z., Jr., Bistatic Radar Cross Sections of Chaff, IEEE Trans. Aerosp. Electron. Syst., AES-20, No. 2 (March 1984).

uniformly distributed orientation angles within the horizontal plane. The chaff cloud is illuminated by a source having arbitrary polarization and arbitrary location relative to the cloud. The average bistatic cross section seen by a receiver of arbitrary location and arbitrary polarization was found and is given by equation (23). The cross section seen by the receiver in the orthogonal receiver polarization is given by equation (24). Both equations (23) and (24) apply to a single dipole. Cloud cross sections result from multiplication of equations (23) or (24) by N, the number of dipoles illuminated by the transmitter and viewed in common by the receiver.

Solutions of equations (23) and (24) require (1) specification of transmit and receive polarizations by defining values of Q_T and Q_R according to examples given, or from the earlier paper¹ in general; (2) specification of geometry parameters θ_1 , θ_2 , and β_ϕ (figure 1); (3) specification of resonant dipole length ($L = \lambda/2$, λ , or $3\lambda/2$ only); (4) determining the cross section parameters σ_1/λ^2 needed from tables 2, 3, or 4 using table 1--depending on choices of Q_T and Q_R some of the c_i may not be required; and (5) computation of equations (23) or (24). The numerical results obtained will equal $\bar{\sigma}/\lambda^2$ or $\bar{\sigma}_x/\lambda^2$. Actual cross sections per dipole require λ be specified.

Equation (23) was used to develop cross sections in section 3 for several specific transmit/receive polarization combinations of linear, tilted linear, and circular polarizations.

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¹ Peebles, Peyton Z., Jr., Bistatic Radar Cross Sections of Chaff, IEEE Trans. Aerosp. Electron. Syst., AES-20, No. 2 (March 1984).

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